SPEE Recommended Evaluation Practice #6 – Definition of Decline Curve Parameters

Background:

The production histories of oil and gas wells can be analyzed to estimate reserves and future oil and gas production rates and to validate results of complex reservoir studies. Because accurate production data are commonly available on most wells, production data analyses can be widely applied.

Decline curve analysis relates past performance of oil and gas wells to future performance, but it requires modification to account for changes in performance due to operating conditions or changes in reservoir behavior.

Decline curves are simply a plot of production rate versus time on semi-log, log-log, or specially scaled paper. The most common plot is semi-log. When the logarithm of producing rate is plotted versus linear time, a straight line often results. This phenomenon is referred to as "exponential decline" or "constant percent decline." If the data plot as a concave upwards curve, a "harmonic" or "hyperbolic decline" model can be used to model the data.

The mathematical equation defining <u>exponential</u> decline has two constants, the initial production rate and the "decline rate." The decline rate is the rate of change of production with respect to time and, for exponential decline, is constant for all time. There are two ways to define the decline rate for exponential decline – "*nominal*" and "*effective.*"

The mathematical equation defining <u>hyperbolic</u> decline has three constants, the initial production rate, the initial decline rate (defined at the same time as the initial production rate), and the "hyperbolic exponent." The decline rate is not a constant, but changes with time, since the data plot as a curve on semi-log paper. The hyperbolic exponent is the rate of change of the decline rate with respect to time, or the second derivative of production rate with respect to time. There are <u>three</u> ways to define the "initial decline rate" for hyperbolic decline – "*nominal*", "*tangent effective*", and "*secant effective*." Figure 1 illustrates the difference between the *tangent effective* and *secant effective* definitions of initial decline rate.

Computer programs usually use the *nominal* form of the equations internally while input and output are usually in terms of *effective* decline. The decline curve equations in terms of *nominal* decline and the equations used to convert from one form of decline to another are as shown below.

Decline Curve Equations (for consistent units)

Exponential decline

Nominal $$D = \frac{\ln\left(q_i / q \right)}{t}$$

Effective: $$D_e = \frac{(q_i - q)}{q_i}$$ for a particular time period, usually 1 year

Effective decline as a function of nominal decline is:

$$D_e = 1 - e^{-D}$$

Nominal decline as a function of effective decline is:

$$D = -\ln(1 - D_e)$$

Hyperbolic decline

Nominal $$D_i = \frac{\left(\frac{q_i}{q} \right)^b - 1}{bt}$$

Tangent Effective $$D_{ei} = \frac{(q_i - q)}{q_i}$$ where q_i and q are read from the <u>tangent</u> line

Secant Effective $$D_{esi} = \frac{(q_i - q)}{q_i}$$ where q_i and q are read from the <u>secant</u> line

Nominal decline as a function of <u>tangent</u> effective decline is:

$$D_i = -\ln(1 - D_{ei})$$

Nominal decline as a function of <u>secant</u> effective decline is:

$$D_i = \frac{(1 - D_{esi})^{-b} - 1}{b}, \quad b \neq 0$$

Where: D = *nominal* exponential decline rate, 1/time

D_i = initial *nominal* decline rate (t=0), 1/time

D_{ei} = initial *effective* decline rate from tangent line, 1/time

D_{esi} = initial *effective* decline rate from secant line, 1/time

q_i = *instantaneous* producing rate at time 0, vol/unit time

q = *instantaneous* producing rate at time t, vol/unit time

t = time

e = base of natural logarithms (2.718…)

b = hyperbolic exponent (Describes how the initial decline rate, D_i, changes with time; varies from 0 to 1 usually. When b = 1, the decline is called harmonic. This exponent is sometimes referred to in the literature as "n")

Discussion:

There are varying opinions over whether the "*effective*" or "*nominal*" decline equation should be used. *Nominal* decline rate can take on any value from negative infinity to positive infinity where negative numbers indicate production rate is increasing rather than decreasing. *Effective* decline rate cannot exceed 1.0 since an ending flow rate of zero results in an effective decline rate of q_i/q_i or 1. This limitation on *effective* decline rate can lead to problems with wells that are experiencing extremely high initial declines such as massive hydraulic fracs in tight gas reservoirs or horizontal wells. It is not unusual to see initial *nominal* decline rates on the order of 30 (3000%) in these cases. The *tangent effective* decline rate corresponding to a *nominal* decline rate of 30 is 0.999999999999906 (thirteen 9's followed by 06.) Since double precision numbers in computers are limited to approximately 15 decimal digits of accuracy, it is impossible to represent any exponential decline rate significantly in excess of 30 using *tangent effective* decline. Some computer programs only allow 8 or fewer digits for decline rate. At 8 digits the maximum *nominal* decline rate that can be represented is approximately 20 (2000%.)

Normally, extremely high initial decline rates are associated with *hyperbolic* decline curves, often with initial values of "b" at or near 2. When "b" is 1 or greater it is possible to use the *secant effective* decline even at extremely high decline rates. If "b" is 2 and the initial *nominal* decline is 100 (10000%) the value of the initial *secant effective* decline rate is 92.946544%. This number is easily represented even in single precision. Table 1 and Figure 2 show the value of *tangent effective* decline (which is independent of "b") and *secant effective* decline for various values of "b."

The *secant effective* decline rate has the additional advantage of being calculated from two rates read from the smooth line through the data – one rate at time 0 (which can be arbitrarily defined) and one rate exactly one year later.

Another factor which must be recognized is that the production rate referred to in the above equations is the *instantaneous* rate at a particular point in time. It is not the average rate for a month or the average rate for a year. In cases where the initial decline rate is high the rate at the beginning of a month may be considerably larger than the average rate during the month.

Any of these methods of defining the future production curve will lead to the correct answer if they are properly applied. The most important thing is clear and consistent communication between the user and the computer programmer.

SPEE Recommended Evaluation Practice:
SPEE recommends that the terminology shown above be used. Further, SPEE recommends that all input and output be clearly labeled using the following names and symbols.

Item	Symbol
Nominal Decline Rate, exponential	D
Initial Nominal Decline Rate, hyperbolic	D_i
Effective Decline Rate, exponential	D_e
Initial Tangent Effective Decline Rate, hyperbolic	D_{ei}
Initial Secant Effective Decline Rate, hyperbolic	D_{esi}

References:

Arps, J.J., (1956). *Estimation of Primary Oil Reserves.* Dallas: Society of Petroleum Engineers.
Fetkovich, M.J. (1980): "Decline Curve Analysis using Type Curves," *J. Pet. Tech.*, (June 1980) 1065-1077.

Figure1 – Decline Rate Definitions for Hyperbolic Decline

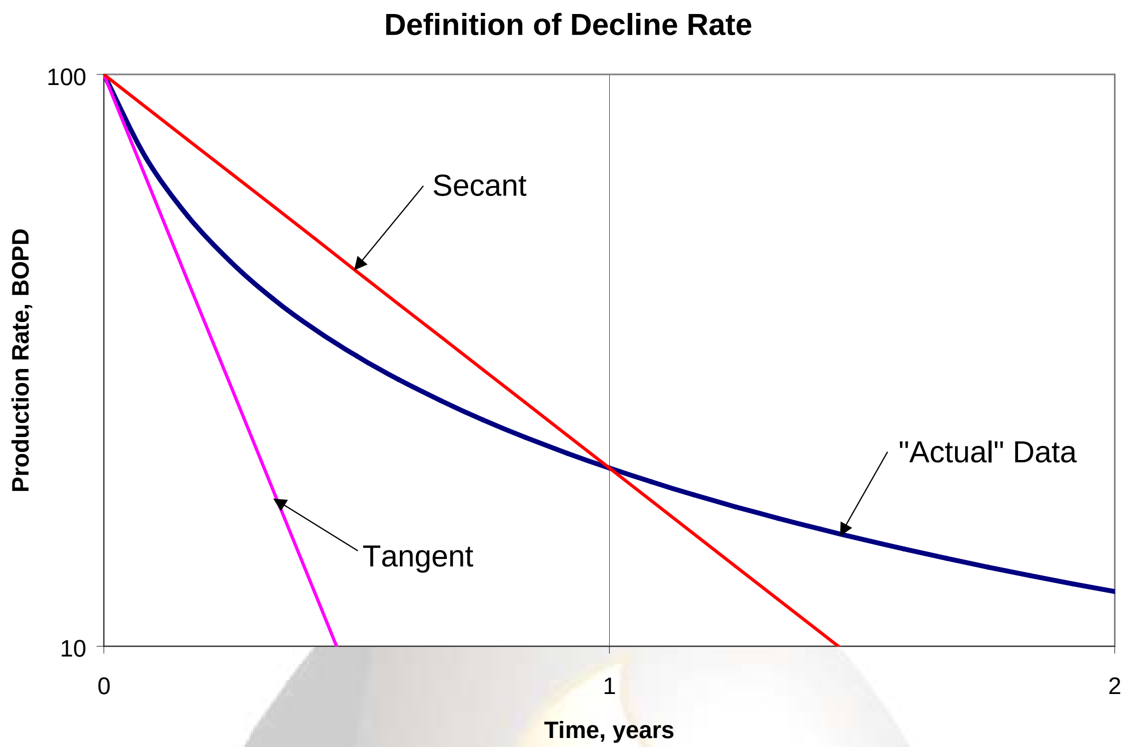

Figure 2 – Effective Decline Rate as a function of Nominal Decline Rate

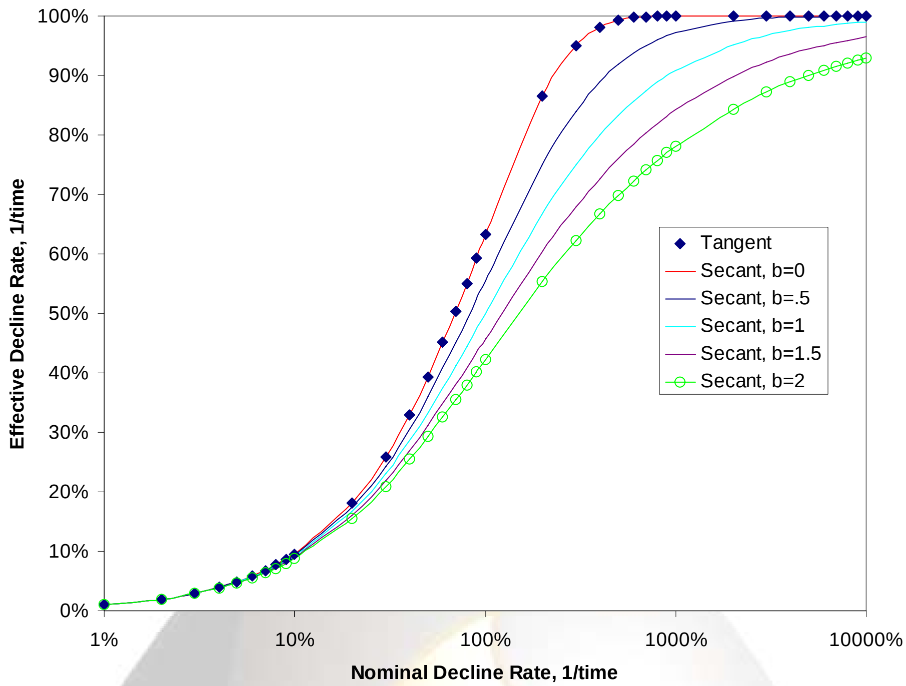

Table 1 - Effective Decline Rate as a function of Nominal Decline Rate

Nominal Decline Rate, 1/year	Effective Tangent Decline Rate, 1/year	Effective Secant Decline Rate, 1/year				
		Value of "b"				
		0	0.5	1	1.5	2
1%	0.99501662508319%	0.995017%	0.992550%	0.990099%	0.987664%	0.985246%
2%	1.98013266932447%	1.980133%	1.970395%	1.960784%	1.951298%	1.941932%
3%	2.95544664514918%	2.955447%	2.933825%	2.912621%	2.891822%	2.871414%
4%	3.92105608476768%	3.921056%	3.883122%	3.846154%	3.810111%	3.774955%
5%	4.87705754992860%	4.877058%	4.818560%	4.761905%	4.706995%	4.653741%
6%	5.82354664157513%	5.823547%	5.740409%	5.660377%	5.583260%	5.508882%
7%	6.76061800940517%	6.760618%	6.648930%	6.542056%	6.439655%	6.341419%
8%	7.68836536133642%	7.688365%	7.544379%	7.407407%	7.276891%	7.152331%
9%	8.60688147287718%	8.606881%	8.427005%	8.256881%	8.095645%	7.942538%
10%	9.51625819640405%	9.516258%	9.297052%	9.090909%	8.896561%	8.712907%
20%	18.12692469220180%	18.126925%	17.355372%	16.666667%	16.046701%	15.484575%
30%	25.91817793182820%	25.918178%	24.385633%	23.076923%	21.941297%	20.943058%
40%	32.96799539643610%	32.967995%	30.555556%	28.571429%	26.899557%	25.464401%
50%	39.34693402873670%	39.346934%	36.000000%	33.333333%	31.138792%	29.289322%
60%	45.11883639059740%	45.118836%	40.828402%	37.500000%	34.812509%	32.580014%
70%	50.34146962085910%	50.341470%	45.130316%	41.176471%	38.032483%	35.450278%
80%	55.06710358827780%	55.067104%	48.979592%	44.444444%	40.882207%	37.982633%
90%	59.34303402594010%	59.343034%	52.437574%	47.368421%	43.425409%	40.238570%
100%	63.21205588285580%	63.212056%	55.555556%	50.000000%	45.711648%	42.264973%
200%	86.46647167633870%	86.466472%	75.000000%	66.666667%	60.314974%	55.278640%
300%	95.02129316321360%	95.021293%	84.000000%	75.000000%	67.905924%	62.203553%
400%	98.16843611112660%	98.168436%	88.888889%	80.000000%	72.672412%	66.666667%
500%	99.32620530009150%	99.326205%	91.836735%	83.333333%	75.990264%	69.848866%
600%	99.75212478233340%	99.752125%	93.750000%	85.714286%	78.455653%	72.264990%
700%	99.90881180344460%	99.908812%	95.061728%	87.500000%	80.372359%	74.180111%
800%	99.96645373720970%	99.966454%	96.000000%	88.888889%	81.912810%	75.746437%
900%	99.98765901959130%	99.987659%	96.694215%	90.000000%	83.182762%	77.058427%
1000%	99.99546000702380%	99.995460%	97.222222%	90.909091%	84.250987%	78.178211%
2000%	99.99999979388460%	100.000000%	99.173554%	95.238095%	89.866514%	84.382624%
3000%	99.99999999999060%	100.000000%	99.609375%	96.774194%	92.210765%	87.196312%
4000%	100.00000000000000%	100.000000%	99.773243%	97.560976%	93.546726%	88.888889%
5000%	100.00000000000000%	100.000000%	99.852071%	98.039216%	94.426548%	90.049628%
6000%	100.00000000000000%	100.000000%	99.895942%	98.360656%	95.057207%	90.909091%
7000%	100.00000000000000%	100.000000%	99.922840%	98.591549%	95.535261%	91.578481%
8000%	100.00000000000000%	100.000000%	99.940512%	98.765432%	95.912324%	92.118896%
9000%	100.00000000000000%	100.000000%	99.952741%	98.901099%	96.218704%	92.567059%
10000%	100.00000000000000%	100.000000%	99.961553%	99.009901%	96.473461%	92.946544%